

Interactive Theorem Proving – 2026-07-17

Exercise 1. *Prove the following statement using a Lean term.*

example

```
(A B C D: Prop)
(h1: (A → B) ∧ (C → D))
(h2: A ∨ C)
: B ∨ D
:= sorry
```

Exercise 2. Prove the following statement using a *Lean term*.

example

```
(α: Type)
(P Q R: α → Prop)
(h1: ∀ a, P a → R a)
(h2: ∀ a, Q a → R a)
(h3: ∃ a, P a ∨ Q a)
: ∃ a, R a
:= sorry
```

Exercise 3. *Expose the underlying motive for the following `match` expression.*

- *Make the match motive explicit.*
- *Add additional matched expressions (scrutinees), if needed by the motive.*
- *Replace any `_` wildcard in patterns with fresh variables or suitable inaccessible patterns.*
- *Ensure the new match expression is valid.*

```
inductive P (x: ℤ): ℤ → Prop
| c1: x > 0 → P x x
| c2: (y: ℤ) → x < 0 → P x (y*y - x)
```

```
example
(Q: ℤ → Prop)
(a b: ℤ)
(h1: P a b)
(h2: ∀ a: ℤ, a > 0 → Q a)
: Q b
:= match h1 with
| .c1 _ => h2 _ (by grind)
| .c2 _ _ => h2 _ (by simp ; nlinarith)
```

