Matriculation ______________ Full name ________________________________

Interactive Theorem Proving – 2026-06-16

Exercise 1. *Prove the following statement using a Lean* <u>*term*</u>.

```
example
  (P Q R S: Prop)
  (h1: (P → Q) ∧ (Q → R))
  (h2: P ∨ (Q ∧ S))
  : R
  := sorry
```

Exercise 2. *Prove the following statement using a Lean* <u>*term*</u>.

```
example
  (α β: Type)
  (P: α → Prop)
  (Q: β → Prop)
  (R: α × β → Prop)
  (h1: (a: α) → P a → (b: β) → Q b → R ⟨a, b⟩)
  (h2: ∃ a, ∃ b, P a ∧ Q b)
  : ∃ c, R c
  := sorry
```

Exercise 3. *Expose the underlying* `motive` *for the following* `match` *expression.*

- *Make the match* `motive` *explicit.*
- *Add additional matched expressions (scrutinees), if needed by the* `motive`*.*
- *Replace any* _ *wildcard in patterns with fresh variables or suitable inaccessible patterns.*
- *Ensure the new match expression is valid.*

Note: the provided match expression might intentionally contain a few type errors for you to fix.

```
inductive P: Bool → ℤ → Prop
| c1: (w: ℤ) → w ≥ 0 → P true (- w - 3)
| c2: (x: ℤ) → P false (x*x + 1)

example
  (Q: ℤ → Prop)
  (b: Bool) (y: ℤ)
  (h1: P b y)
  (h2: ∀ a: ℤ, a < 0 → Q a)
  : Q (if b then y else -y)
  := match h1 with
  | .c1 _ _ => h2 _ (by grind)
  | .c2 _   => h2 _ (by simp ; nlinarith)
```